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ABSTRACT

This document provides teaching guidelines and student material for a unit intended for use in 12th grade algebra classes. Time allotment is from four to six hours of classroom time. The objective of this capsule is to teach students how to solve compound interest problems using arithmetic, logorithms, and calculators. Prerequisites for the unit are second year algebra, a working knowledge of logorithms and exponential equations, and an available calculator. The student material is divided into three sections. Section I focuses on solving compound interest problems, deriving the compound interest formula, and analyzing several savings accounts to determine which pays the most. Section II emphasizes using the compound interest formula, while Section III concentrates on solving compound interest problems for the rate of interest for the time the money was in the account. Seven activity sheets provide word problems relating to compound interest and a comparative analysis chart. Three supplementary activities focus on computer program problem solving and further comparative analysis. The teacher's guide provides a list of needed materials, advance preparation information, keys to activities and tests, a bibliography, and copies of all worksheets and assessments. (CK)

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COMPOUND INTEREST IS AS EASY AS PI π

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Teacher's Guide

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ESEA Title IV
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Easton, KS 66020

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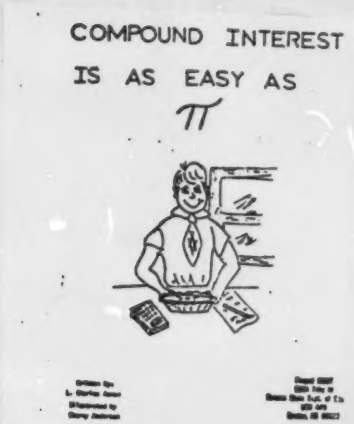

PACKAGE RESUME FORM

Publication Date--September, 1979

Author--Charles Auman

Title--Compound Interest Is As Easy as π

Level/Subject--12th Grade Algebra

Abstract--This capsule is designed to teach students how to solve compound interest problems using arithmetic, logorithms, and calculators. It is self-contained except for the scientific calculator that must be available for student use.

This capsule is designed for high school seniors taking a math analysis course or an equivalent course. The student must have successfully completed second year algebra and must have a working knowledge of logorithms and exponential equations in order to complete this capsule.

The student should be able to complete the activities in this capsule in four to six hours.

GENERAL INSTRUCTIONS

Materials List:

Provided:
- Student Capsule
- Worksheets
- Pre-Test
- Post-Test

1. You will need the above materials for each student doing the capsule.

2. The capsule is self-contained except the student should have a calculator available which has scientific functions, hopefully, e^x, y^x, and log functions.

3. Quest Activity 3 is a comparative study of savings institutions. You may want to require it. If you require it you may want to have ads available from local newspapers.

4. The student must have a working knowledge of logarithms, simple interest, and a calculator. They hopefully will have had an introduction to e and natural logs.

5. There are two quest activities using a computer, if one is available and the student can program it.

Name Key Date _____

Circle the best answer to each question.

1. Carol has had $300 in a savings account earning 6% simple interest for one year. How much is in her account now?

   a. $300

   (b.) $318

   c. $306

   d. $312

2. John put $100 in a savings account 1 year ago. The account pays 5% interest compounded twice a year. What amount is in the account now?

   a. $100

   b. $105

   c. $105.25

   (d.) $105.06

3. If you have $100 in an account paying 8% interest compounded quarterly, how much money will you have in 3 years?

   (a.) $126.82

   b. $124

   c. $124.82

   d. $126

4. If you have $100 in a savings account which pays 6% interest compounded monthly, how long will it take to have $200 in the account?

   a. 11 years and 5 months

   b. 11 years and 7 months

   (c.) 12 years

   d. 12 years and 6 months

5. What is interest ?

variable

6. What is compound interest?

variable

MATCHING

d	7. P	a.	number of compounding periods
c	8. R	b.	interest
f	9. T	c.	rate
e	10. A	d.	principal
b	11. I	e.	amount
a	12. N	f.	time in years

WORKSHEET #1

Name Key Date _____

Work the following problems using only arithmetic. Show the work that is done as in the example (in the student capsule). Give this to your teacher when you are finished.

1. John has $300 in a savings account that pays 6% interest compounded monthly. How much money will he have in the account in one year?

1. $(300)(.06)(1/12) = 1.5$
2. $(301.5)(.06)(1/12) = 1.51$
3. $(303.01)(.06)(1/12) = 1.52$
4. $(304.53)(.06)(1/12) = 1.52$
5. $(306.05)(.06)(\frac{1}{12}) = 1.53$
6. $(307.58)(.06)(\frac{1}{12}) = 1.54$
7. $(309.12)(.06)(\frac{1}{12}) = 1.55$
8. $(310.67)(.06)(\frac{1}{12}) = 1.55$
9. $(312.22)(.06)(\frac{1}{12}) = 1.56$
10. $(313.78)(.06)(\frac{1}{12}) = 1.57$
11. $(315.35)(.06)(\frac{1}{12}) = 1.58$
12. $(316.93)(.06)(\frac{1}{12}) = 1.58$

$318.51

2. First National Bank of Smalltown has a special savings account that pays 7% compounded quarterly if you save at least $500 and leave it in for one year. What would be in the account after one year if the minimum was saved?

$(500)(.07)(\frac{1}{4}) = 8.75$

$(508.75)(.07)(\frac{1}{4}) = 8.90$

$(517.65)(.07)(\frac{1}{4}) = 9.06$

$(526.71)(.07)(\frac{1}{4}) = 9.22$

$535.93

WORKSHEET #2

Name Key Date _____

Work the following problems using logarithms and the formula for solving compound interest problems. You will need log tables or if your teacher permits, a calculator with log functions. Show how you do the problems.

1. Sue put $200 into a savings account 4 years ago. If the account paid 8% interest compounded quarterly, how much is in the account now?

$P = 200$
$r = .08$
$t = 4$
$n = 4$

$A = 200\left(1 + \frac{.08}{4}\right)^{4 \cdot 4}$
$A = 200(1.02)^{16}$
$A = 200(1.37)$
$A = 274$

$\log (1.02)^{16} = 16 \log 1.02$
$= 16(.0086)$
$= .1376$
$\log^{-1} .1376 = 1.37$

2. Assume there are 3 savings institutions within one mile of where you live. You have $100 to invest. Which would give you the best return on your money in 2 years?

a) Henry's Savings & Loan pays 5% interest compounded 5 times a year.

$A = 100\left(1 + \frac{.05}{5}\right)^{5 \cdot 2} = 100(1.01)^{10}$
$= 100(1.10)$
$= 110$

$\log^{-1}[(\log 1.01)10] =$
$\log^{-1}(.0043)10 =$
$\log^{-1}(.043) = 1.10$

Calculator Value $110.46
log Tables Value $110.

b) Joan's State Bank pays 4½% interest compounded daily.

$A = 100\left(1 + \frac{.045}{365}\right)^{365 \cdot 2} = 100$

Calculator value $109.43
log Tables Value $100.

c) Workers Credit Union pays 5¼% interest compounded yearly.

$A = 100\left(1 + \frac{.0525}{1}\right)^{2} = 110.25$

Calculator value $110.78
log Tables Value $110.25

WORKSHEET #3

Name Key Date _____

Work the following problems using a calculator. Indicate the main function you use, i.e. y^x, log x, lnx, or arithmetic functions. Before using the calculator, substitute the values into the formula.

1. Union Federal Savings & Loan Company pays 6% compounded daily on their passbook account. Judy has $175 in the account. How much will she have in 3 years?

$A = 175\left(1 + \frac{.06}{365}\right)^{365 \cdot 3} = \$209.52$

2. You have $350 to put into a savings account. The bank down the street pays 5¼% compounded monthly. A bank across town pays 5 3/4% compounded quarterly. In which bank would you deposit your money and why?

$A_1 = 350\left(1 + \frac{.0525}{12}\right)^{12}$

$A_1 = \$368.83$

$A_2 = 350\left(1 + \frac{.0575}{4}\right)^{4}$

$A_2 = 370.56$

3. Professor Higgens put $500 into a savings account 10 years ago. For 5 years the account paid 5% interest compounded semiannually, then for 3 years it paid 5½% interest compounded monthly. For the last 2 years it has paid 6% interest compounded daily. How much money does Professor Higgens have in her account now?

$A_1 = 500\left(1 + \frac{.05}{2}\right)^{10}$

$A_1 = 640.04$

$A_2 = A_1\left(1 + \frac{.055}{12}\right)^{36}$

$A_2 = 754.58$

$A_3 = A_2\left(1 + \frac{.06}{365}\right)^{365 \cdot 2}$

$A_3 = 850.82$

$\$850.82$

WORKSHEET #4

Name Key Date _____

Find the amount at the end of the first year for $1 invested at 100% interest compounded:

1) annually $2

2) semiannually $2.25

3) quarterly $2.44

4) monthly $2.61

5) weekly $2.69

6) daily $2.71

7) hourly $2.72

8) every second $2.72

9) instantaneously (estimate) $2.7182818 = e
$2.72

WORKSHEET #5

Name Key Date _____

Solve the following problems using e^x either on a calculator, or using the the table in the appendix. Show your work and final equation.

1. Assume you have $100 in an account paying 7% interest compounded instantaneously. How much money would be in the account in one year?

$100\, e^{.07} = 107.25$

2. Henry can put his $250 into two banks. Which one would earn the most money?

a) Hometown Community Bank which pays 6% interest compounded instantaneously. $250\, e^{.06} = 250(1.0618) = \$265.45$

b) Community National Bank which pays 6¼% compounded monthly. $250\left(1 + \frac{.0625}{12}\right)^{12} = \$266.08$

3. Jean put $375 in Smalltown Savings & Loan Co. 2 years ago. They paid 5% interest compounded instataneously. How much does she have in her account now?

1 year $375\, e^{.05} = 375(1.0513) = 394.875$

2 year $394.875\, e^{.05} = 394.875(1.0513) = \$415.13$

WORKSHEET #6

Name Key Date _____

Answer the following problems. Remember to show your work.

1. Your bank sends you a statement that says you have $127.04 in your savings account. You remember putting $100 in it 4 years ago, and you know the compounding period is monthly but you do not remember the rate of interest. Find the rate of interest.

$$127.04 = 100\left(1+\frac{r}{12}\right)^{48} \qquad 1.2704^{\frac{1}{48}} = 1+\frac{r}{12}$$
$$1.2704 = \left(1+\frac{r}{12}\right)^{48}$$
$$\log 1.2704 = 48 \log\left(1+\frac{r}{12}\right)$$
$$\frac{1}{48}\log 1.2704 = \log\left(1+\frac{r}{12}\right)$$
$$\log 1.2704^{\frac{1}{48}} = \log 1+\frac{r}{12}$$

2. There are two banks in your town. One pays 5% interest compounded monthly and the second one pays 5% interest compounded daily. If you put $100 in each account, how long will it take for each account to have $125 in it? (b) How long will it take for you to have $300 total?

$$125 = 100\left(1+\frac{.05}{12}\right)^{12t} \qquad 125 = 100\left(1+\frac{.05}{365}\right)^{365t}$$
$$\log 1.25 \overset{\log}{=} \left(1+\frac{.05}{12}\right)^{12t} \qquad \frac{\log 1.25}{365\left(\log\frac{365.05}{365}\right)} = t$$
$$\log 1.25 = 12t\left(\log\left(1+\frac{.05}{12}\right)\right)$$
$$\frac{\log 1.25}{12\log\left(\frac{12.05}{12}\right)} = t \qquad = t$$
$$= t$$

b) $300 = 100\left(1+\frac{.05}{12}\right)^{12t} + 100\left(1+\frac{.05}{365}\right)^{365t}$

3. You get a statement in the mail that says you have $268.05 in your savings account. You remember putting $250 in it a year ago. Find the probable interest rate if you do not know the compounding period. (Make an estimate and defend it.)

POST-TEST

Name Key Date _____

Circle the best answer to each question.

1. What interest is earned on $300 saved for 3 years at 9% simple interest?

(a.) $81 $i = 300(3)(.09)$

b. $381

c. $388.50

d. $88.50

2. How long will it take to double your money in an account paying 9% interest compounded monthly?

$2P = P(1 + \frac{.09}{12})^{12t}$

$2 = (1 + \frac{.09}{12})^{12t}$

$2 = (1.0075)^{12t}$

$\frac{\log 2}{12 \log 1.0075} = t$

$\frac{.3010}{12(.0032)} = 7.8$ years

7 years 10 months

a. 2 years

b. 5 years and 2 months

(c.) 7 years 10 months

d. 12 years

3. John had $575 in a savings account 5 years ago. For 3 years it earned 5½% interest compounded quarterly. For the last two years it earned 7% compounded daily. How much is in the account today?

$A = [575(1 + \frac{.055}{4})^{12}](1 + \frac{.07}{365})^{(365.2)}$

$A = \$779.208$

or rounding

$575(1.014)^{12}(1.0002)^{730}$

$\$786.20$

Answers will vary depending on method of computation.

a. $667.40

b. $661.31

c. $741.75

(d.) $779.21

4. What is the effective annual rate for 7% interest compounded instantaneously?

$e^{.07} = 1.0725$

(a.) 7¼%

b. 7%

c. 8%

d. 7½%

5. What is interest?

variable

6. What is compound interest?

variable

7. What does "P" stand for? Principal

8. What does "i" stand for? interest

9. What does "n" stand for? Compounding periods per year

10. What does "a" stand for? amount

11. What does "r" stand for? rate of interest

12. What does "t" stand for? time in years

APPENDIX

This appendix contains copies of all worksheets and assessments. These copies may be used to make your own classroom quantity copies. To use, unstaple the sides of the teacher's guide and remove the worksheet and assessment pages. Restaple the remaining pages of the teacher's guide.

Name _____ Date _____

RECORD SHEET

scores	comments
_____ Pre-Test	_____
_____ Worksheet 1	_____
_____ Worksheet 2	_____
_____ Worksheet 3	_____
_____ Worksheet	_____
_____ Post-Test	_____
_____ Quest Activity 1	_____
_____ Quest Activity 2	_____
_____ Quest Activity 3	_____

PRE-TEST

Name _____ Date _____

Circle the best answer to each question.

1. Carol has had $300 in a savings account earning 6% simple interest for one year. How much is in her account now?

   a. $300

   b. $318

   c. $306

   d. $312

2. John put $100 in a savings account 1 year ago. The account pays 5% interest compounded twice a year. What amount is in the account now?

   a. $100

   b. $105

   c. $105.25

   d. $105.06

3. If you have $100 in an account paying 8% interest compounded quarterly, how much money will you have in 3 years?

   a. $126.82

   b. $124

   c. $124.82

   d. $126

4. If you have $100 in a savings account which pays 6% interest compounded monthly, how long will it take to have $200 in the account?

   a. 11 years and 5 months

   b. 11 years and 7 months

   c. 12 years

   d. 12 years and 6 months

PRE-TEST (con't)

5. What is interest ?

_____

_____

6. What is compound interest?

_____

_____

MATCHING

_____ 7. P	a.	number of compounding periods
_____ 8. R	b.	interest
_____ 9. T	c.	rate
_____ 10. A	d.	principal
_____ 11. I	e.	amount
_____ 12. N	f.	time in years

WORKSHEET #1

Name _____ Date _____

Work the following problems using only arithmetic. Show the work that is done as in the example (in the student capsule). Give this to your teacher when you are finished.

1. John has $300 in a savings account that pays 6% interest compounded monthly. How much money will he have in the account in one year?

2. First National Bank of Smalltown has a special savings account that pays 7% compounded quarterly if you save at least $500 and leave it in for one year. What would be in the account after one year if the minimum was saved?

WORKSHEET #2

Name _____ Date _____

Work the following problems using logarithms and the formula for solving compound interest problems. You will need log tables or if your teacher permits, a calculator with log functions. Show how you do the problems.

1. Sue put $200 into a savings account 4 years ago. If the account paid 8% interest compounded quarterly, how much is in the account now?

2. Assume there are 3 savings institutions within one mile of where you live. You have $100 to invest. Which would give you the best return on your money in 2 years?

   a) Henry's Savings & Loan pays 5% interest compounded 5 times a year.

   b) Joan's State Bank pays 4½% interest compounded daily.

   c) Workers Credit Union pays 5¼% interest compounded yearly.

WORKSHEET #3

Name _____ Date _____

Work the following problems using a calculator. Indicate the main function you use, i.e. y^x, log x, lnx, or arithmetic functions. Before using the calculator, substitute the values into the formula.

1. Union Federal Savings & Loan Company pays 6% compounded daily on their passbook account. Judy has $175 in the account. How much will she have in 3 years?

2. You have $350 to put into a savings account. The bank down the street pays 5¼% compounded monthly. A bank across town pays 5 3/4% compounded quarterly. In which bank would you deposit your money and why?

3. Professor Higgens put $500 into a savings account 10 years ago. For 5 years the account paid 5% interest compounded semiannually, then for 3 years it paid 5½% interest compounded monthly. For the last 2 years it has paid 6% interest compounded daily. How much money does Professor Higgens have in her account now?

WORKSHEET #4

Name _____ Date _____

Find the amount at the end of the first year for $1 invested at 100% interest compounded:

1) annually _____

2) semiannually _____

3) quarterly _____

4) monthly _____

5) weekly _____

6) daily _____

7) hourly _____

8) every second _____

9) instantaneously (estimate) _____

WORKSHEET #5

Name _____ Date _____

Solve the following problems using e^x either on a calculator, or using the the table in the appendix. Show your work and final equation.

1. Assume you have $100 in an account paying 7% interest compounded instantaneously. How much money would be in the account in one year?

2. Henry can put his $250 into two banks. Which one would earn the most money?

   a) Hometown Community Bank which pays 6% interest compounded instantaneously.

   b) Community National Bank which pays 6¼% compounded monthly.

3. Jean put $375 in Smalltown Savings & Loan Co. 2 years ago. They paid 5% interest compounded instataneously. How much does she have in her account now?

WORKSHEET #6

Name _____ Date _____

Answer the following problems. Remember to show your work.

1. Your bank sends you a statement that says you have $127.04 in your savings account. You remember putting $100 in it 4 years ago, and you know the compounding period is monthly but you do not remember the rate of interest. Find the rate of interest.

2. There are two banks in your town. One pays 5% interest compounded monthly and the second one pays 5% interest compounded daily. If you put $100 in each account, how long will it take for each account to have $125 in it? (b) How long will it take for you to have $300 total?

3. You get a statement in the mail that says you have $268.05 in your savings account. You remember putting $250 in it a year ago. Find the probable interest rate if you do not know the compounding period. (Make an estimate and defend it.)

WORKSHEET #7

Name _____ Date _____

1. Fill out the following chart using information you have found in newspapers, magazines, or by calling savings institutions. If no newspapers or magazines have the information at school, do the work at home or the public library.

Name of Institution	Interest Rate	Compounding Period	1 yr. interest on $100	distance from your home	guaranteed or not
a.					
b.					
c.					
d.					
e.					
f.					
g.					

2. Which savings account pays the best ? _____

3. Does the highest rate pay the most money? _____

4. Which one is closest to your home? _____

5. Which institution is easiest for you to do business with? _____

6. Do your parents have an account at one of them? (b) Is it the best paying? _____

7. What other things should you consider before choosing a savings institution?

POST-TEST

Name _____ Date _____

Circle the best answer to each question.

1. What interest is earned on $300 saved for 3 years at 9% simple interest?

   a. $81

   b. $381

   c. $388.50

   d. $88.50

2. How long will it take to double your money in an account paying 9% interest compounded monthly?

   a. 2 years

   b. 5 years and 2 months

   c. 7 years 10months

   d. 12 years

3. John had $575 in a savings account 5 years ago. For 3 years it earned 5½% interest compounded quarterly. For the last two years it earned 7% compounded daily. How much is in the account today?

   a. $667.40

   b. $661.31

   c. $741.75

   d. $779.21

4. What is the effective annual rate for 7% interest compounded instantaneously?

   a. 7¼%

   b. 7%

   c. 8%

   d. 7½%

POST-TEST (con't)

5. What is interest?

_____

_____

6. What is compound interest?

_____

_____

7. What does "P" stand for? _____

8. What does "i" stand for? _____

9. What does "n" stand for? _____

10. What does "a" stand for? _____

11. What does "r" stand for? _____

12. What does "t" stand for? _____

COMPOUND INTEREST IS AS EASY AS PI

π

Student Material

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Illustrated by
Sherry Anderson

Project COST
ESEA Title IV
Kansas State Dept. of Education
USD 449
Easton, KS 66020

Check it out!

INTRODUCTION

Compound interest makes savings accounts much more "interesting". Simple interest may be easily figured, but it is not as good a deal for the consumer.

This capsule is intended to show how you can figure compound interest no matter what the compounding period or the rate of interest. You will find that compound interest can even be related to other parts of mathematics.

You will be solving compound interest problems which may confront you later in life. Once you have discovered the easiest method of solving the compound interest problems, you will be solving some problem situations consumers face daily.

It is expected that you will have a background in mathematics. You will need to know how to use logarithms and you will hopefully have a calculator with scientific functions, i.e. log, ln or y^x.

You are to do all the activities in the capsule except the Quest Activities which are optional. Your teacher may give some special directions involving some of the activities.

OBJECTIVES

Upon completion of this capsule, you will be able to:

- solve compound interest problems for the amount of money in the account given the rate, time and period.
- derive the compound interest formula from the formula i=prt.
- analyze several savings accounts to determine which pays the most interest to the consumer.
- show the derivation of e using the compound interest formula.
- solve the compound interest problems for the interest rate or the time.
- solve compound interest problems that have an instantaneous compounding period.

Before beginning this capsule, get the pre-test from your teacher, complete it and return it to your teacher for checking.

$$A = P\left(1 + \frac{R}{N}\right)^{NT}$$

SECTION I

OBJECTIVES

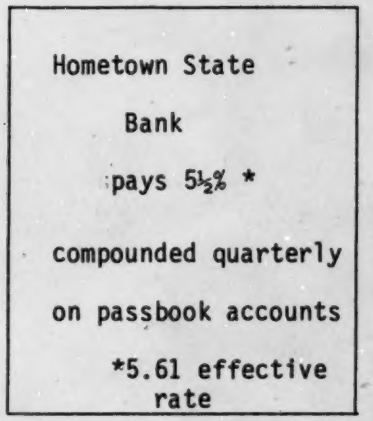
Upon completion of this section you will be able to:

* 1. solve compound interest problems for the amount of money in the account given, the rate, time and period.

* 2. derive the compound interest formula from the formula $i = prt$.

* 3. analyze several savings accounts to determine which pays the most interest to the consumer.

Advertisements such as this one for Hometown State Bank tell the rate of interest as well as the compounding period. The effective rate is the simple interest rate which would produce the same income.



To find the simple interest on \$100 at 5½% interest for 1 year. Use the formula i = prt. i = 100(.055)1 = 5.50. The interest is \$5.50 and the amount in the account would be \$105.50

To figure the compound interest each compound period must be taken individually.

PERIOD 1	\$100	@	5½% for ¼ yr	=	\$1.375
PERIOD 2	\$101.375	@	5½% for ¼ yr	=	\$1.394
PERIOD 3	\$103.769	@	5½% for ¼ yr	=	\$1.413
PERIOD 4	\$104.182	@	5½% for ¼ yr	=	\$1.432

TOTAL interest earned in one year is	\$5.614
Amount in the account after one year is	\$105.61
Effective rate (simple interest)	\$5.61

This process can be used to derive the formula for finding compound interest.

LET A = amount of money in the account

P = principal or amount of money invested

r = rate of interest in decimal form

t = time in years

n = number of compounding periods in one year

Period 1 $A = P + P\frac{rt}{n} = P\left(1 + \frac{rt}{n}\right)$

Period 2 $A = \left(P + P\frac{rt}{n}\right) + \left(P + P\frac{rt}{n}\right)\left(\frac{rt}{n}\right)$

$= P\left(1 + \frac{rt}{n}\right) + P\left(\frac{rt}{n} + \left(\frac{rt}{n}\right)^2\right)$

$= P\left(1 + \frac{2rt}{n} + \left(\frac{rt}{n}\right)^2\right)$

$= P\left(1 + \frac{rt}{n}\right)^2$

Period 3 following from Period 2

$= P\left(1 + \frac{rt}{3}\right)^3$

Period 4 continuing

$A = P\left(1 + \frac{rt}{n}\right)^4$

$100\left(1 + \frac{.055\frac{1}{4}}{4}\right) = 101.375$

$101.375 + 101.375\left(1 + \frac{.055\frac{1}{4}}{4}\right)$

$100\left(1 + \frac{(.055)\frac{1}{4}}{4}\right)^3 = 104.182$

$100\left(1 + \frac{(.055)\frac{1}{4}}{4}\right)^3 = 104.182$

$100\left(1 + \frac{(.055)\frac{1}{4}}{4}\right)^4 = 105.61$

In the Hometown State Bank example, n = 4, because interest was compounded quarterly. Since time was for one year.

$$A = \$100\ (1 + \frac{(.055(1)}{4})^4 \quad \text{or}$$

$$A = \$100\ (1 + \frac{.055}{4})^4$$

(1) Solving for A

$$A = 105.61$$

If an individual saved for 2 years instead of one year, twice as many compounding periods are taken. The account accumulates

(2) $A = \$100\ (1 + .055\frac{2}{8})^8$ or

$$A = 111.54$$

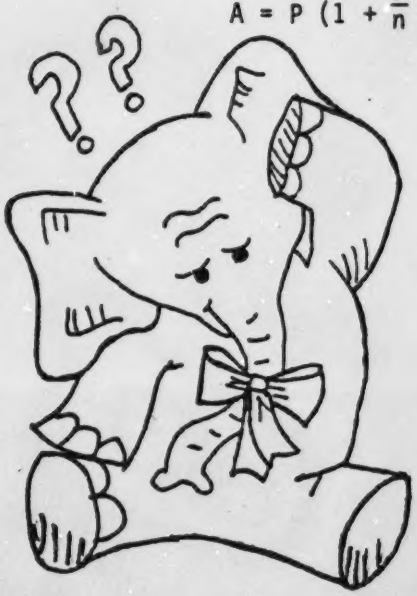
In the example $P = \$100$, $r = .055$, $t = 2$, $n = 4$, $2n = 8$, $nt = 8$

Substituting properly into equation (2)

$A = P\ (1 + r\ \frac{2}{2n})^{nt}$ or

$A = P\ (1 + \frac{r}{n})^{nt}$

(3)



We will be working the compound interest problems using arithmetic (boring), logarithms (not bad), and calculators (easy as π). Let's start with the arithmetic approach to see the development as well as appreciate the easier methods.

ARITHMETIC METHOD

Find the amount in account for 6 months at 6% interest compounded monthly with a beginning balance of \$100.

$P = \$100$

$r = .06$

$n = 12$

$t = \frac{1}{2}$

$$A = P\left(1 + \frac{r}{n}\right)^{tn}$$

$$A = 100\left(1 + \frac{.06}{12}\right)^{(\frac{1}{2})(12)}$$

$$A = 100\,(1 + .005)^{6}$$

$$A = 100\,(1.005)\,(1.005)\,(1.005)\,(1.005)(1.005)\,(1.005)$$

$$A = 100\,(1.0304)$$

$$A = \$103.04$$

SIMPLE INTEREST

$i = prt$ $\qquad i = (100)\ (.06)\ (.5)$

$i = 6(.5) = \$300$

$A = P + i = \$100 + 3.00 = \$103.00$

The difference between simple and compound interest is only 4¢.

Compound interest is of great advantage to long term savers.

Activity 1

Ask your teacher for Worksheet #1. Do the problems using only arithmetic. Do not use a calculator. The problems are kept fairly simple, and there are not too many. Hand your paper in to your teacher when you have finished.

Working compound interest problems using only arithmetic can be tedious. Using a calculator and/or logarithms helps to shorten the work time.

TO LOGARITHMS

LOGARITHM METHOD

You will need log tables or if your teacher will allow it, a calculator with log functions. Log tables can be found in your math book.

Find the amount in an account with $275 in it at 6% for 5 years compounded monthly.

$$A = \$275 \left(1 + \frac{.06}{12}\right)^{(12 \cdot 5)}$$

$$A = \$275 \, (1 + .005)^{60}$$

$$A = \$275 \, (1.005)^{60}$$

$\log (1.500)^{60} = 60 \log 1.005$

$= 60 \, (.0022)$

$= .1320$

$(1.005)^{60} = 1.35$

$\log^{-1}(\log (1.005)^{60} = \log^{-1} (.1320)$

$1.005^{60} = 1.35$

$A = \$275 \, (1.35) = \$371.25$

$i = prt$

$i = \$275 \, (.06) \, (5) = 275 \, (3) = 82.5$

$P + i = A$

$275 + 82.5 = \$357.50$

There is quite a bit of difference after five years between simple and compound interest!

Activity 2

Get Worksheet #2 from your teacher. Do the problems on the worksheet. Use logarithms for at least the powers. Show your work even if you use a calculator. Hand this worksheet in to your teacher when you have finsihed.

CALCULATOR METHOD

For this section a calculator with log function or a power function such as y^x is needed.

Calculator (with y^x button)

Find the Amount in the account that had \$350 in it for 8 years paying 5½% compounded quarterly.

$A = 350\ (1 + \frac{.055}{4})^{(4 \cdot 8)}$

$A = 350\ (1 + .01375)^{32}$

$A = 350\ (1.01375)^{32}$ enter 1.01375, y^x, 32=

$A = 350\ (1.5480598)$

$A = \$541.82$

If you want to use the calculator all the way through you could use the formula in this manner:

$A = P \times (1 + (r \div n))\ y^x\ (n \times t)$

or if you do not have a parenthesis key:

nt = nxt

A = nxt sto r ÷ n + 1 = y^x RCL = x P

A =n x t sto r ÷ n + 1 = y^x RCL = x P

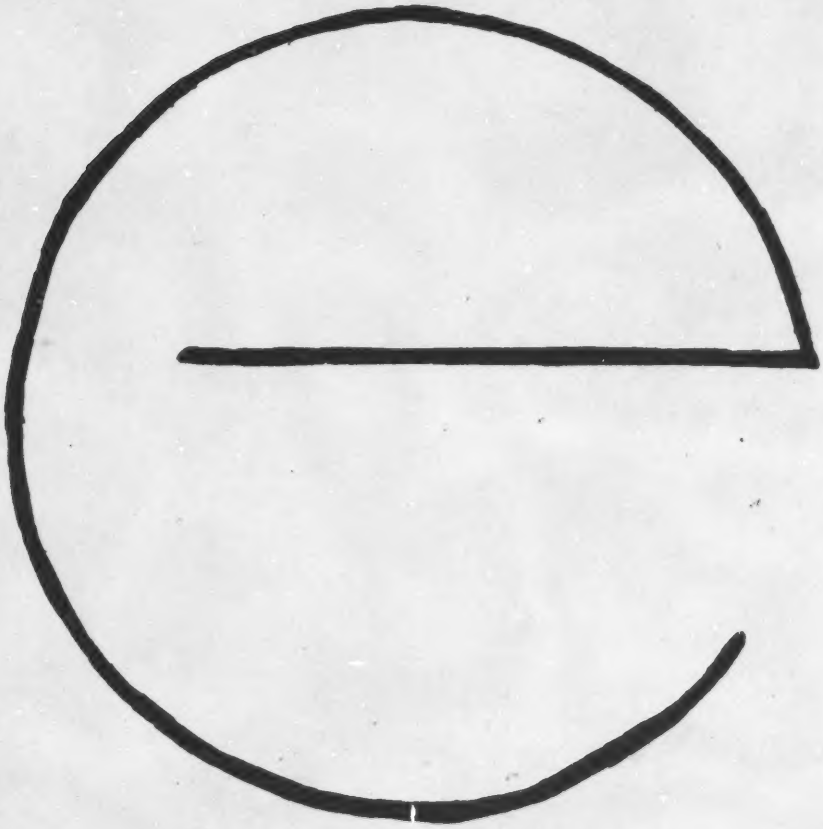
A = r ÷ n + 1 = log x n x t 10^x x P

Activity 3

Work the problems on Worksheet #3. Use your calculator, but show the work as in the example.

SECTION 2

Compounding Instantaneously?



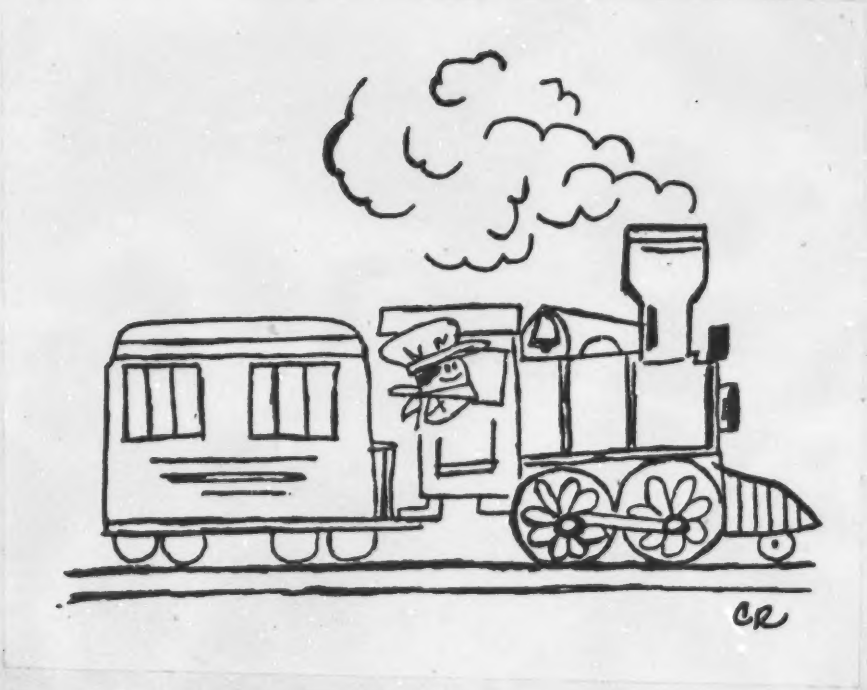
SECTION II

Objectives

Upon completion of this section you will be able to:

* 1. show the development of "e" using the compound interest formula

* 2. solve compound interest problems that have an instantaneous compounding period.



Activity 4

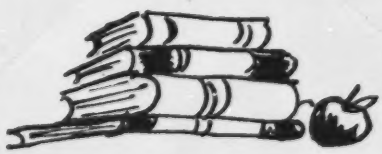
Get Worksheet #4 from your teacher. Do the problems on Worksheet #4 using any method you choose. Keep this worksheet until you are finished with Section 2.

As the compounding period is shortened the amount in the account is increased. Since compounding instantaneously is the shortest the compounding period could be, it will give us the maximum amount in the account. From Activity 4 we will have \$1 @ 100% interest compounded instantaneously for one year gives us approximately \$2.71828. When all the figures in Activity 4 are rounded to the nearest cent the most you can have at the end of one year is \$2.72 which can be earned by compounding hourly.

Mathematically speaking compounding this problem instantaneously approaches a number called the limit. This number is called e in honor of the Swiss Mathmetician Leonhard Euler who proved that the limit exists.

So, the definition of e is:

(4) $e = \lim (1 + \frac{1}{n})^n = 2.718281828459045$



Notice this equation is very similar to the equation for finding the amount.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

If P = 1, r = 1, and t = 1, we would have e as n gets very large.

Example 4 (using the definition of e)

Find the amount A in the bank earning 6% interest compounded instantaneously if there was \$1 in it one year ago, then:

$$A = 1\left(1 + \frac{.06}{365}\right)^{365} = \left(\left(1 + \frac{1}{\frac{365}{.06}}\right)^{\frac{365}{.06}}\right)^{.06}$$

if $n = \frac{365}{.06}$ then

$$A = \left(\left(1 + \frac{1}{n}\right)^{n}\right)^{.06}$$

$$A = e^{.06}$$

NOTE: looking $e^{.06}$ up in the table or use a calculator

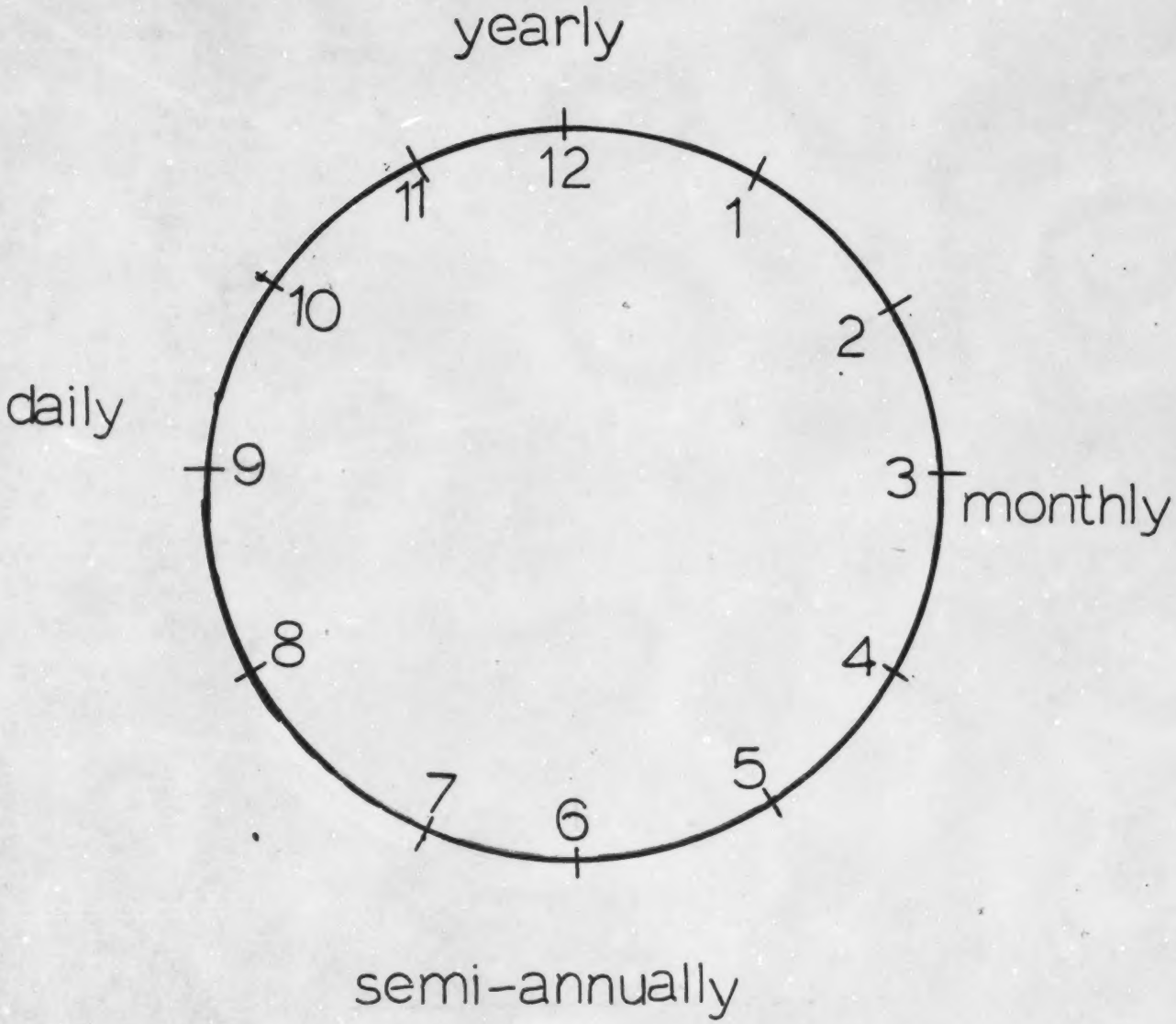
$e^{.06} = \$1.0618$

Activity 6

Get Worksheet #5 from your teacher. Do the problems. Hand the worksheet in to your teacher when you are finished.

SECTION 3

Rate & Time



SECTION III

Objectives

Upon completion of this section you will be able to:

* 1. solve compound interest problems for the rate of interest or the time the money was in the account

Not all compound interest problems are to find the amount. Upon completion of this section you will be able to:

Objective: Use logarithmic properties to solve compound interest problems for rate and time.

To solve compound interest problems for the rate, start with the formula and solve for r.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

using logarithmic properties,

$$\log A = \log P + nt \log\left(1 + \frac{r}{n}\right)$$

$$\log^{-1}\left(\frac{\log A - \log P}{nt}\right) = \log^{4}\left(\log\left(1 + \frac{r}{n}\right)\right)$$

$$n\left[\log^{-1}\left(\frac{\log A - \log P}{nt}\right) - 1\right] = r \qquad \text{or}$$

$$n\left[\log^{-1}\left(\frac{A}{pnt}\right) - 1\right] = r$$

It is possible to learn formula (5), but it may be easier to remember formula (3) and solve for the specific r using the same process used in the derivation.

Example 5

John has had \$300 in his savings account for one year. He got a statement telling him he had \$381.50 in the account and knows it is compounded monthly. What is the rate of interest paid on the account.

$$318.50 = \$300\left(1 + \frac{r}{12}\right)^{12}$$

$$\frac{318.50}{300} = \left(1 + \frac{r}{12}\right)^{12}$$

$$\sqrt[12]{\frac{318.50}{300}} = 1 + \frac{r}{12}$$

$$\sqrt[12]{\frac{318.50}{300}} - 1 = \frac{r}{12}$$

$$12\left(\sqrt[12]{\frac{318.50}{300}} - 1\right) = r$$

using a calculator

$$.059 = r$$

(knowing the accuracy of the calculator, it might be 6%)

Joan wants to know how long it will take her \$250 to become \$300 if it is in an account paying 7% interest compounded daily.

$$300 = 250\left(1 + \frac{.07}{365}\right)^{365t}$$

$$\frac{300}{250} = \left(\left(1 + \frac{1}{\frac{365}{.07}}\right)^{\frac{365}{.07}}\right)^{.07t}$$

$$1.2 = (e^{.07t})$$

$$1.2 = e^{.2} = e^{.07t}$$

$$.2 = .07t$$

$$\frac{20}{7} = t$$

or t = 2 years 313 days

The formula $\frac{A}{P} = e^{rt}$ or

$\ln \frac{A}{P} = rt$ or

$\frac{1}{r}\left(\ln \frac{A}{P}\right) = t$ could be used

Activity 6

Get Worksheet #6 from your teacher. Do the problems and turn the worksheet in to your teacher when you have finished.

Quest Activities

1. Write and run a computer program to solve compound interest problems for the amount in the account. Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$

2. Write and run a computer program solving compound interest problems for the rate and/or the time.

3. Use Worksheet #7 to investigate the different rates and compounding periods of banks, savings & loan companies, and credit unions in your area.

APPENDIX

x	e^{x}	e^{-x}
0.00	1.0000	1.0000
0.04	1.0408	0.9608
0.05	1.0513	0.9512
0.06	1.0618	0.9418
0.07	1.0725	0.9324
0.08	1.0833	0.9231
0.09	1.0942	0.9139
0.10	1.1052	0.9048